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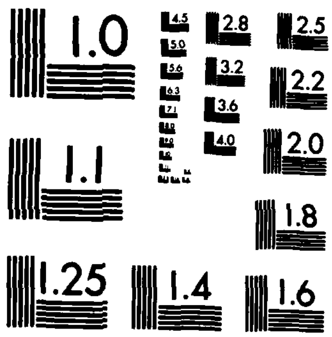

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B. J. S. Barnard[†], W. G. Pritchard[†,*] and D. G. Provis[†]

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ABSTRACT

A description is given of some experiments made to examine the possible trapping of surface waves by a submerged cylindrical sill in an otherwise uniform ocean. Theories by Longuet-Higgins (1967) and Renardy (1981) indicate that, at certain dangerous frequencies, nearly resonant conditions may obtain, leading to unusually large wavefields over the sill.

These 'resonances' have been sought experimentally by examining two functionals of the wavefield over the sill to see if they should become unusually large near certain of the 'dangerous' frequencies. The results show no manifestation whatsoever of the resonances. Possible reasons for the discrepancies are considered.

It has, however, been observed that, at certain locations over the sill, the waves were amplified by a factor of four or five times their magnitude in the 'ocean', irrespective of the frequency in a certain interval. This could be an important consideration for the siting of rigs on prominent topographic features of the ocean bed.

AMS (MOS) Subject Classifications: 76B15

Key Words: Water waves, trapping

Work Unit Number 2 (Physical Mathematics)

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[*]Part of this work was done at the Mathematics Research Center, University of Wisconsin-Madison, 53706.

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EXPERIMENTS ON WAVE TRAPPING BY A SUBMERGED CYLINDRICAL ISLAND

B. J. S. Barnard[†], W. G. Pritchard[†,*] and D. G. Provis[†]

1. INTRODUCTION

The purpose of the present paper is to describe some experiments concerning the possible trapping of waves by certain topographic features of an ocean bed. The importance of such trapping was first appreciated by Ursell (1951,1952) who showed that the Stokes edge wave over a plane beach (e.g. see Lamb 1932, p 447) corresponds to just one of a set of discrete eigensolutions of the associated mathematical problem. The amplitudes of the wavemodes corresponding to these discrete eigenvalues have an exponentially decaying structure at large distances from the shore and the energy contained in them is finite. For this reason it is common to consider the waves as being trapped in the zone near the shore. But the very fact that the waves are localized near the shore means that they cannot be excited directly by the forcing effects of a wavemaker sited at a large distance from the shore. Thus, to realize these modes in the laboratory Ursell (1952) forced the motions in the zone near the shore by oscillating the side walls of his channel. He also suggested that it might be possible to generate them through a nonlinear mechanism, and this was subsequently shown to be the case by Galvin (1965).

For the mathematical problem considered by Ursell (1951, 1952) the eigenvalues are all real but, when slightly more complicated geometric situations are examined, the operators associated with the mathematical problem often are not self adjoint (cf. Lozano & Meyer 1976). Thus we

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can, in general, expect any discrete eigenvalues arising in these situations to be complex, the imaginary part corresponding to a 'leakage' or radiation of energy to the far field. Nevertheless, it might be anticipated that, under certain conditions (in particular in situations bearing similarities to those in which edge waves are possible), the imaginary parts of some of the discrete eigenvalues would be small and we could expect such waves to have properties closely related to those of the perfectly resonant situation. This has indeed proved to be the case in some specific problems. For example, it has been shown for a submerged seamount (Longuet-Higgins 1967, Renardy 1981) and for conical islands with a shore (Shen, Meyer & Keller 1968, Meyer 1971, Lozano & Meyer 1976) that there are discrete eigenvalues with very small leakages. The fact that these wavemodes can potentially have a large response in certain regions of the ocean (near the important topographical features) and yet 'leak' energy to the far field is of considerable interest. For, in contrast to the edge-wave situation, the leakage allows for the possibility of exciting these 'nearly resonant' modes directly (without the need for a nonlinear coupling mechanism) by means of a wavetrain generated at large distances from the island.

Only two serious attempts appear to have been made so far to investigate experimentally this concept of wave 'resonance' in open bodies of water. One of these, a forerunner of the present paper, was a study by Provis (1975) (and see Provis 1976) who, for the cases of a conical island and a submerged cylindrical sill, considered the motions arising near these topographic features from the action of a wavemaker at one side of a square tank. The other study (Pite 1977) concerned the motions above a submerged cylindrical sill. Even though both these studies were undertaken with considerable care, neither experiment produced any prima facie evidence to suggest the presence of resonant-like resonse. Pite measured wave amplitudes along a radial line normal to the direction of propagation of the incident wavetrain and found, for three different frequencies, that his results were described fairly well by a theoretical model incorporating an allowance for the effects of damping. It was inferred from this agreement that, under suitable conditions, the near resonances predicted by Longuet-Higgins (1967) should be evident. But Renardy (1981) has shown that the frictional effects were not correctly incorporated into the theory used by Pite, and the agreement may therefore have been fortuitous.

One complication of experimental work in this area is the recognition of a 'resonance' should it occur. So, in order to establish a procedural basis on which to locate 'resonances', we decided to measure some functionals associated with the wavefield, (certain real-valued properties of the wavefield as a function of frequency,) and then to count as resonance any range of frequencies for which a functional assumes abnormally large values. For example, if we were to examine the maximum wave amplitude over the entire ocean (relative to that of the incident wavetrain), we should refer to any frequency for which this quantity takes on unusually large values as being a 'resonant' frequency. Not only are these functionals useful in characterising any resonances, they also provide a convenient way of summarizing the results for comparison with theory, as we shall indicate below.

The particular experiment to be described in this paper concerns the possible trapping of waves by a submerged cylindrical sill in an open ocean of uniform depth. The initial work of Provis (1976), in which trapping near a conical island with a shore was considered, did not reveal the predicted resonances. A possible explanation of this result is that the resonances were overwhelmed by dissipation, which should have been substantial, in the zone near the shore of the island, (cf. Mahony & Pritchard 1980). So, not only did the submerged sill offer the attraction of being relatively insensitive to dissipative effects (a feature that has subsequently been confirmed in a calculation by Renardy 1981) but also the explicit solutions given by the theory of Longuet-Higgins (1967) provided a framework for comprehending the experimental results. We therefore made measurements of both the maximum wave amplitude, as a function of frequency, and of the 'energy' functional. There was no evidence from these results to suggest the occurrence of the dramatic resonances predicted theoretically.

In an attempt to find a possible reason for this disparity, Renardy (1981) relaxed the shallow-water assumption used by Longuet-Higgins and established a complete eigenfunction expansion for the (linear) problem. These more exact calculations confirmed the 'resonances' of the Longuet-Higgins theory, indicating even larger responses over a slightly narrower bandwidth, but gave the surprising result that the locations of the resonant frequencies were shifted substantially from those predicted by the shallow-water model. Renardy also showed that dissipative effects on the plateau section of the sea mount should have had only a very minor influence on the wave amplitudes. Thus again the empirical results were at variance with the theoretical predictions.

The reason why there is such a large disparity between our experimental results and the theoretical model is not understood. As is the case for all laboratory experiments, there are several respects in which the experimental conditions did not match properly the mathematical problem studied by Renardy (1981) and it is conceivable that any of these could have played an important modifying role in the structure of the wavefield. We have therefore given a fairly detailed description of the experimental apparatus and a table summarising the main experimental results for the two functionals that have been measured.

2. EXPERIMENTAL APPARATUS

The tank used for these experiments was constructed by placing a large sheet of polythene on the floor of the laboratory and raising the edges of the sheet over concrete walls. A plan view of the tank, together with its dimensions, is given in figure 1. Located near three sides of the tank were plane beaches of slope approximately 1:10. These beaches each sloped towards the cylindrical sill near the centre of the tank. A large proportion of the fourth side of the retaining wall of the tank consisted of a plank of laminated timber. This plank, which was used to generate waves in the tank, was hinged near the floor; at the centre, near the top of the plank, it was attached to a mechanism used to force the wavemotion, as described below. With the plank placed behind the polythene sheet, no leakage around the ends of the wavemaker was possible.

Located near the centre of the tank was an upright 'cylinder', referred to in figure 1 as a sill, of mean diameter 100.4 cm and height 13.8 cm. The sill consisted of a disk of perspex with strengthening webbing on its underside and a circumferential skirt extending down to approximately 5 mm from the floor of the tank. The structure was mounted on three feet the heights of which were adjustable to allow for levelling of the upper surface. The levelling was done with the tank filled with water as follows. A finely pointed cone was attached to the end of a micrometer screw and the micrometer was clamped to a small tripod. The tripod was then placed on the upper surface of the sill and the micrometer was slowly screwed down until the tip of the

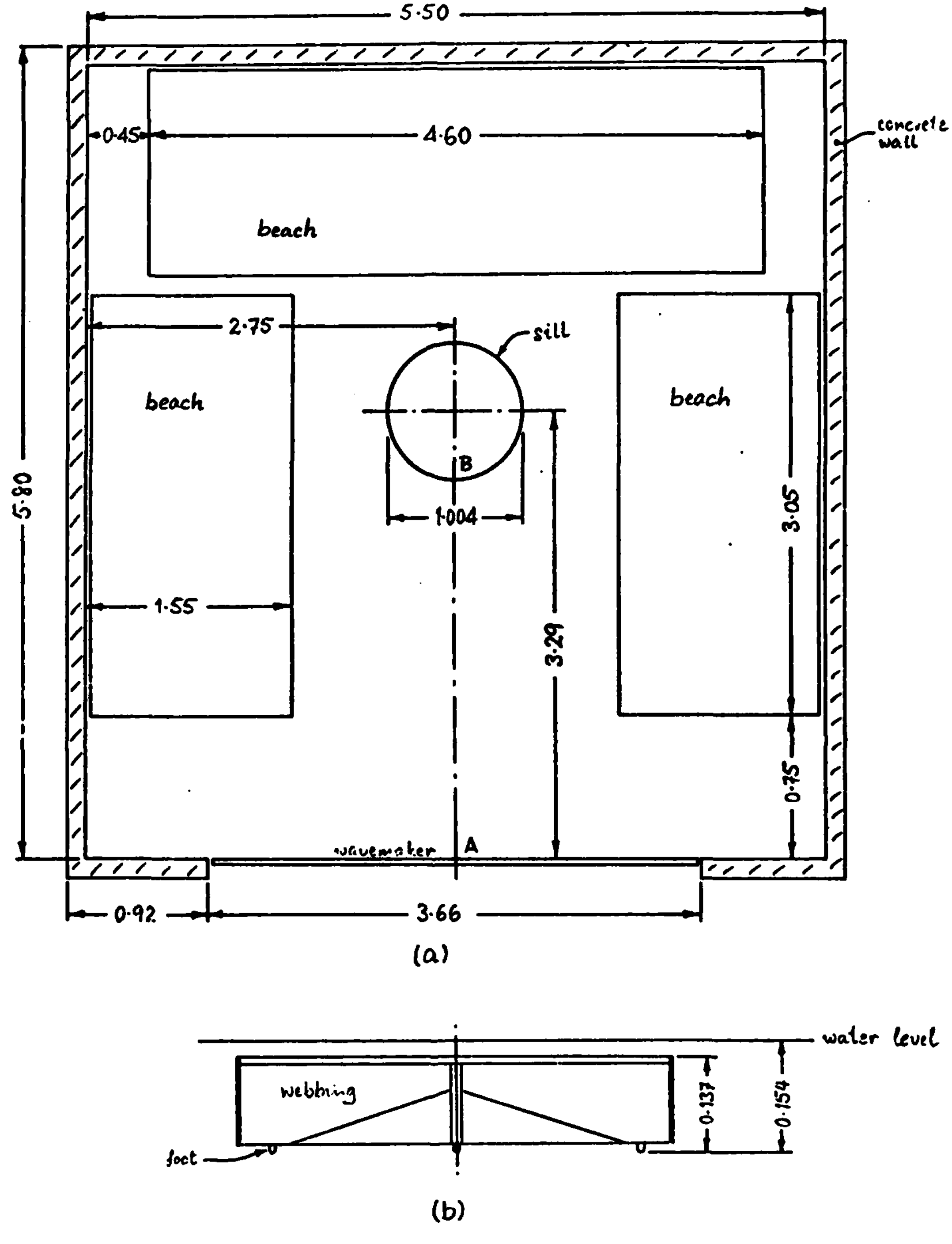

Figure 1. (a) A plan view of the tank used for the experiments.
(b) A sketch of the sill. (All dimensions are in metres.)

cone just broke the water surface, the micrometer setting at this height providing a measure of the distance between the upper surface of the sill and the water surface. After the sill had been suitably levelled a systematic survey indicated that its upper surface deviated from a horizontal plane by no more than 0.2 mm. This 'depth gauge' was also used to set the water level before each experiment: the tripod was placed at a specific location on the sill and the water level was adjusted until it was within 0.02 mm of a nominal value.

Because of the large surface area involved in this experiment, contaminants could easily accumulate at the water surface (originating both in the air and the water itself) and unless special precautions were taken, the resulting surface films could give unacceptable levels of wave damping. We did not make direct quantitative tests of this feature, but we were guided by our experiences from other surface-wave experiments carried out in the same laboratory (see, for example, Barnard & Pritchard, 1972, Barnard, Mahony & Pritchard 1977, Mahony & Pritchard 1980, and Bona, Pritchard & Scott, 1981). This experience suggests that a skimming of the surface just before an experiment leaves it clean enough for about an hour before the build up of contaminants causes significant changes in the wave motions. The skimming of the surface films was brought about by allowing a small overflow into a circular weir placed in the tank; the outflow over the weir, drawing mainly from the surface layers, pulled off the surface films. This cleaning procedure ran continually whenever an experiment was not in progress. Then, just before starting a new set of measurements, the weir was raised and the flux of water into the tank was cut off. A reference measurement of the wavefield was made at the beginning and end of each experiment to establish the 'drift' of the wavefield; this was less than 5% for all the results given here.

The beaches

As indicated above, three sides of the tank were flanked by beaches extending from just above the water level at the tank wall and sloping at 1:10 down to the floor. The beaches were made of aluminium and were covered with a coat of priming paint which was rough enough to retain a film of water beyond the natural shoreline. (This apparently can help the wave absorption properties of the beach, see Mahony & Pritchard 1980.) For the experiments to be described the forcing period ranged between 0.75 s and 1.25 s. The theory of Mahony & Pritchard (1980) for wave absorption by beaches should give a fairly good estimate of the reflection properties of these beaches. This theory indicates that a normally-incident wave of period 0.75 s should have about 98% of its enery dissipated on the beach and a wave of period 1.25 s should have approximately 94% of its energy absorbed by the beach.

The wavemaker

The theory of Longuet-Higgins (1967) indicates that the response of a given mode will be abnormally large over only a very small bandwidth of frequencies. For example, in the experiments to be described, the period of the wavemotions was of the order of 1 s and the 'bandwidth' of one of the abnormally large responses, at about this frequency, is predicted to be approximately 0.03 s. Such narrow bandwidths place severe constraints on the frequency control of the wavemaker, requiring variations in the wave period of, say, less than 3×10^{-3} s, a constraint that has to be maintained for a time span of about an hour. Frequency stability of this kind is not usually possible with most electric motors and we therefore decided to base the frequency source for the wavemaker on a crystal oscillator which was stable to within 5 parts in 10^7.

Over the course of the experiment two wavemaker drives were employed. In the early stages a wavemaker of the kind described in Barnard et al. (1977) was used, but eventually we changed to a different, more convenient device. Pulses from the crystal oscillator were used to control a (400 - pole) stepping motor, the shaft of which was coupled through a crank to the paddle. The latter device gave good direct control over the wavemaker amplitude as well as providing the frequency stability needed for the experiment.

Measuring technique

Measurements of the wave amplitudes were made using proximity transducers placed near the surface of the water. The principle of the instrument is that the transducer forms one plate of a capacitor, the water surface being the other plate. Then, by determining the capacitance it is possible to infer the distance of the water surface from the electrode. The electronics associated with the instrument (Wayne Kerr B731B Proximity Meter) provided an output voltage that varied linearly with the distance between the two electrodes, for separations up to about half the diameter of the transducer. The frequency response of the system ranged from d.c. to approximately 1 kHz.

The output voltage from the proximity gauge was relayed to an ultraviolet chart recorder, to provide a permanent record of the displacement of the water surface at the position of the transducer. For a sinusoidal response, which was usually the case in these experiments, the total displacement was taken to be a measure of twice the wave amplitude. The wave amplitudes arising in this study were of the order of 0.05 mm in the main body of the tank and 0.25 mm over the sea mount. The response over the sill was found to increase in direct proportion to the amplitude of the wavemaker, even at much larger amplitude levels than those used here. The capacitance probes used to measure these waves had a diameter of 11 mm.

Measurements of the wavefield were made by suspending the capacitance transducers from a moveable bridge that spanned the wavetank. For the systematic survey over the island, seven transducers were mounted on an arm that could be rotated about a vertical axis aligned with the centre of the island. The transducers were located at radii of 3.6, 10.8, 18.0, 25.2, 32.4, 39.6 and 46.8 cm, and measurements were made over one half of the island at 19 angular locations, each separated by 10°. (Preliminary tests indicated that the wavefield over the island was nearly symmetric about the diameter normal to the wavemaker.)

The (angular) displacement of the wavemaker was also determined through the use of a capacitance transducer.

3. EXPERIMENTAL RESULTS

3.1 Notation

Let the origin for a cylindrical-polar coordinate system in the horizontal plane be at the centre of the (submerged) island and at the undisturbed position of the free surface. Denote a point $\underset{\sim}{x} = (r,\theta)$ in this plane by its radial (r) and angular (θ) coordinates, where $\theta = 0$ is taken along the line normal to the wavemaker and in the direction from the centre of the island to the wavemaker. Here r (and $\underset{\sim}{x}$) are assumed to be dimensionless quantities, being scaled in terms of the island radius a. Let the undisturbed depth of water over the island be d, and that in the 'ocean' outside the island be D. Define the period of the wavemaker to be T and write $\sigma = 2\pi/T$. Let the vertical displacement of the water surface from its equilibrium position at the point $\underset{\sim}{x}$ be $\tilde{u}(\underset{\sim}{x},t)$, and define a dimensionless wave amplitude $u(\underset{\sim}{x},t) = \tilde{u}(\underset{\sim}{x},t)/\tilde{U}$, where $\tilde{U}$ is a measure of the magnitude of the wavetrain incident on the island.

We suppose that u is of the form

$$u(\underset{\sim}{x},t) = U(\underset{\sim}{x})\exp\{i(\sigma(t) + \phi(\underset{\sim}{x}))\} ,$$

where t is time and ϕ is a phase.

Write $U_{ij} = U(r_i,\theta_j)$, where $r_i = \{\frac{1}{14} + \frac{1}{7} i : i = 0,1,\ldots,6\}$ and $\theta_j = \{j\pi/18 : j = 0,1,\ldots,18\}$, and define functionals U_m and E of U as follows:

$$U_m(\sigma) = \max\{U_{ij} : i = 0,\ldots, 6 ; j = 0,\ldots,18\} ,$$

and $$E(\sigma) = \sum_{i=o}^{6} \sum_{j=o}^{18} w_{ij}\, U_{ij}^2\, r_i\, \Delta r\, \Delta\theta ,$$

where w_{ij} represents a 'weight', chosen according to some quadrature rule, associated with the point (r_i,θ_j) and $\Delta r = \frac{1}{7}$ and $\Delta\theta = \pi/18$. Here the w_{ij} were chosen thus: for each i, $w_{i(1)} = w_{i(18)} = 1$, $w_{i(2k-1)} = 2$ for $k = 1,\ldots,9$ and $w_{i(2k)} = 4$ for $k = 1,\ldots,8$. (These weights correspond to using Simpson's rule as the quadrature formula to approximate the θ-integration and to using the mid-point rule to approximate the integration over r.) The quantity U_m is therefore an approximation to the largest wave amplitude above the island (at the frequency σ) and E is a measure of the (square of the) energy in the wavefield above the island.

3.2 The wavefield over the sill

All the experimental results described here were made with D = 15.4 cm and d = 1.75 cm. Measurements of U_{ij} were obtained at a number of discrete values of T, beginning at T = 0.7500 s and incrementing T by 0.010 s to a value of 1.2000 s. Measurements of U_m only were then obtained with T increased by the same increment up to a value of 1.2500 s. The size of these increments was chosen to respect the bandwidths estimated from the theory of Longuet-Higgins (1967).

(i) The amplitude distribution

Through an interpolation of the measurements U_{ij} we have constructed level sets for U = 0.5 and U = C for C = 1,2,3,.... Some typical examples of the contours are given in figure 2. All these maps have the common feature that there is a zone of relatively large amplitudes near the line $\theta = 0$ and that the maximum wave amplitudes were found either on, or very close to, the line $\theta = 180^{\circ}$. In fact, the largest observed value of U_{ij} at a given period was either $U_{(3)(18)}$ or $U_{(2)(18)}$ at all but seven of the frequencies used, and in those cases the maximum response was found at one of the locations adjacent to these two. The actual locations of $\max\{U_{ij}\}$ are given below in table 2. Further discussion of the wavefield depicted in figure 2(e) is given below.

(ii) The functionals U_m and E

A graph depicting the measurements of $U_m(T)$ and E(T) is given in figure 3. Also shown in this figure are the theoretical values for E(T) given by the theory of Renardy (1981). The reference wave amplitude used to scale the experimental results in this figure was directly related to the amplitude, s, of the wavemaker. Thus, U was

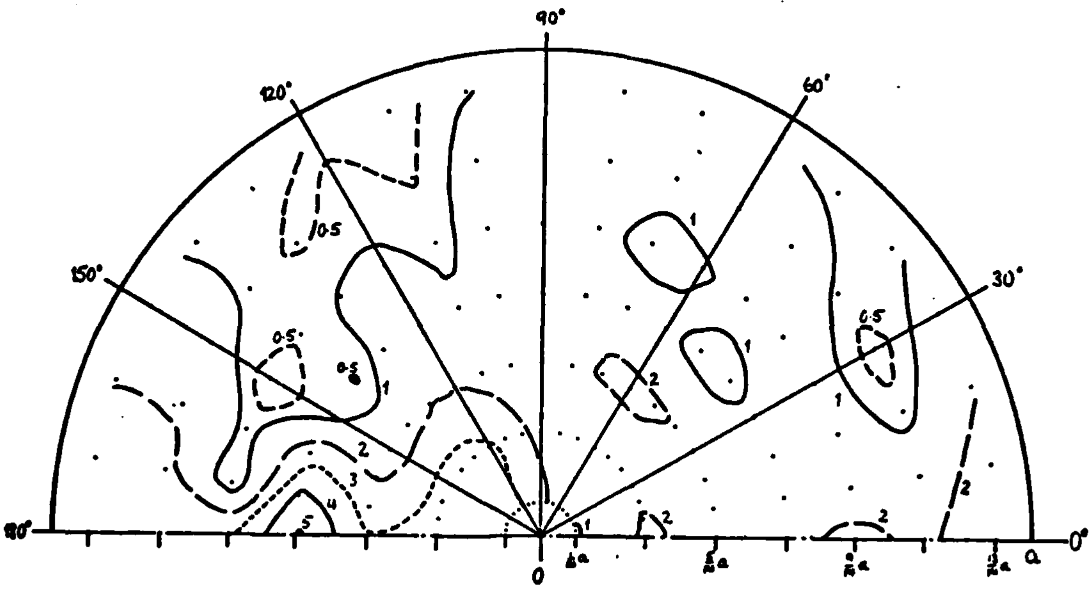

Figure 2a (T = 0.8300s)

Figure 2. Level sets for the amplitude field $u(\underset{\sim}{x}) = \tilde{u}(\underset{\sim}{x})/\tilde{U}$.
(a) T = 0.8300s, $\tilde{U}$ = 0.233 mm; (b) T = 0.9500s, $\tilde{U}$ = 0.226 mm; (c) T = 1.0200s, $\tilde{U}$ = 0.284 mm; (d) T = 1.0800s, $\tilde{U}$ = 0.139 mm; (e) T = 1.1800s, $\tilde{U}$ = 0.208 mm. The grid of dots indicate the positions at which measurements were made. The values of u are indicated near each contour. The code is:
---: u = 0.5; ———: u = 1; ——·——: u = 2; ·······: u = 3;
——: u = 4; — —: u = 5; ------: u = 6; ———: u = 7.

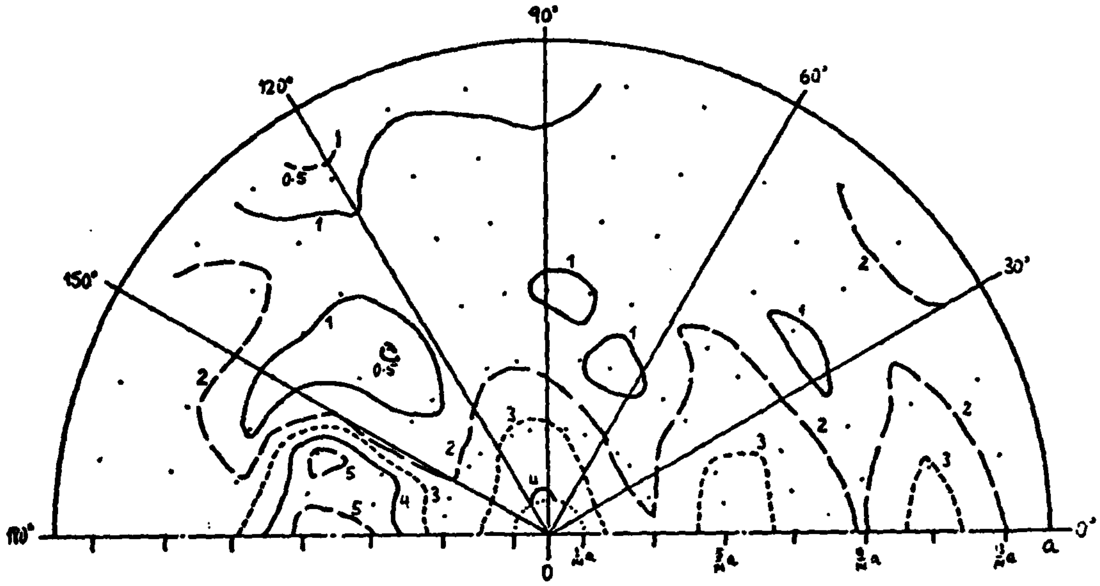

Figure 2b (T = 0.9500s)

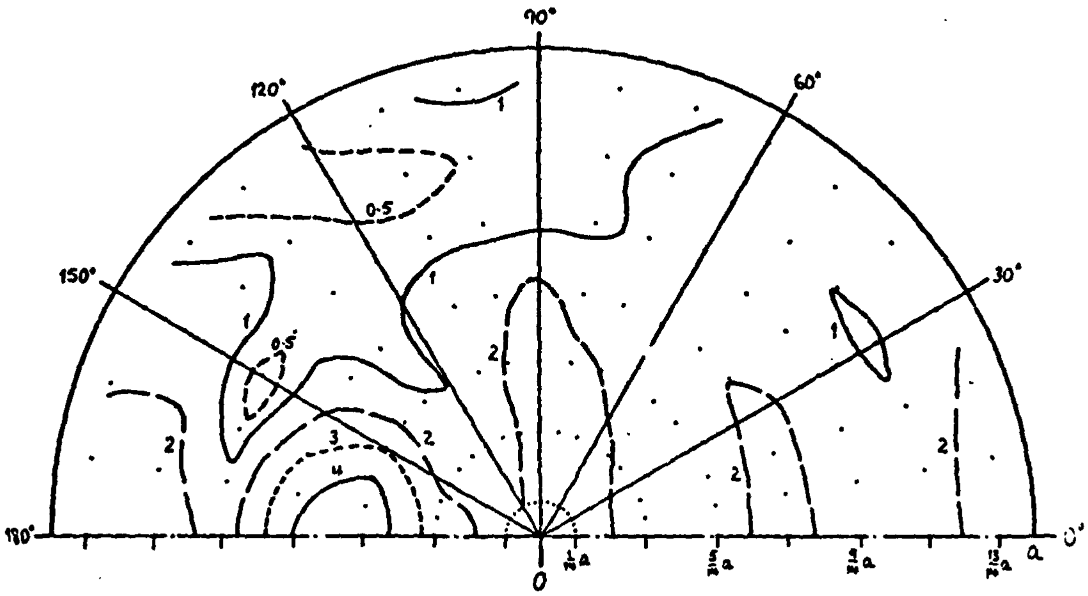

Figure 2c (T = 1.0200s)

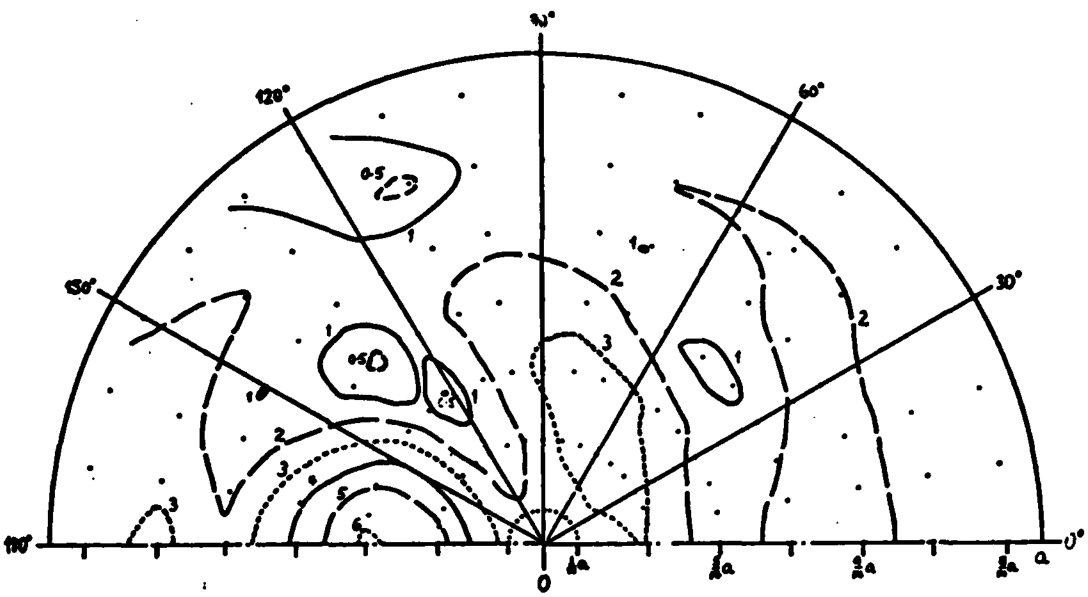

Figure 2d (T = 1.0800s)

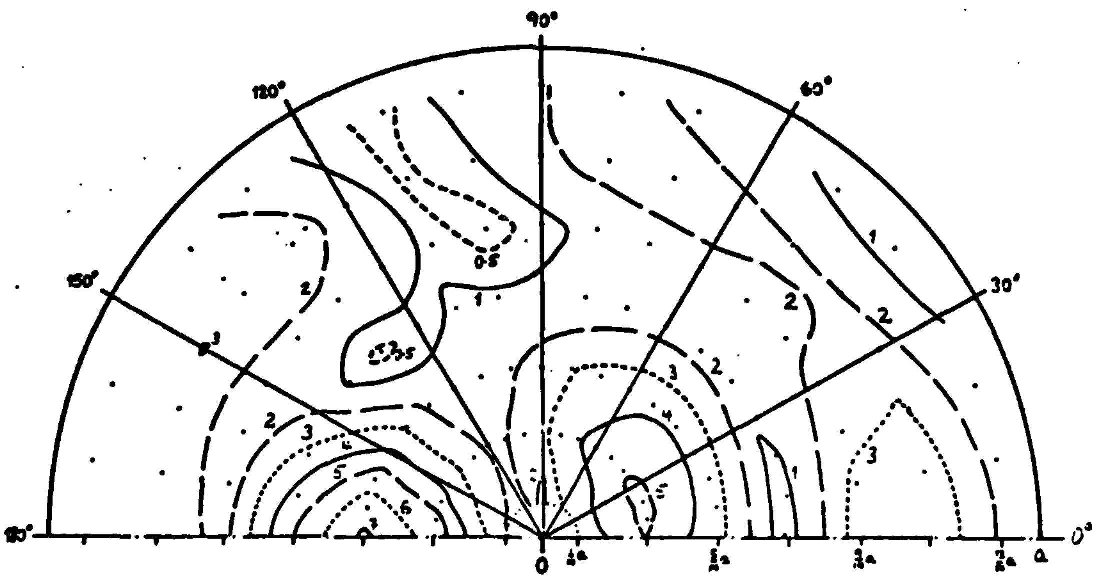

Figure 2e (T = 1.1800s)

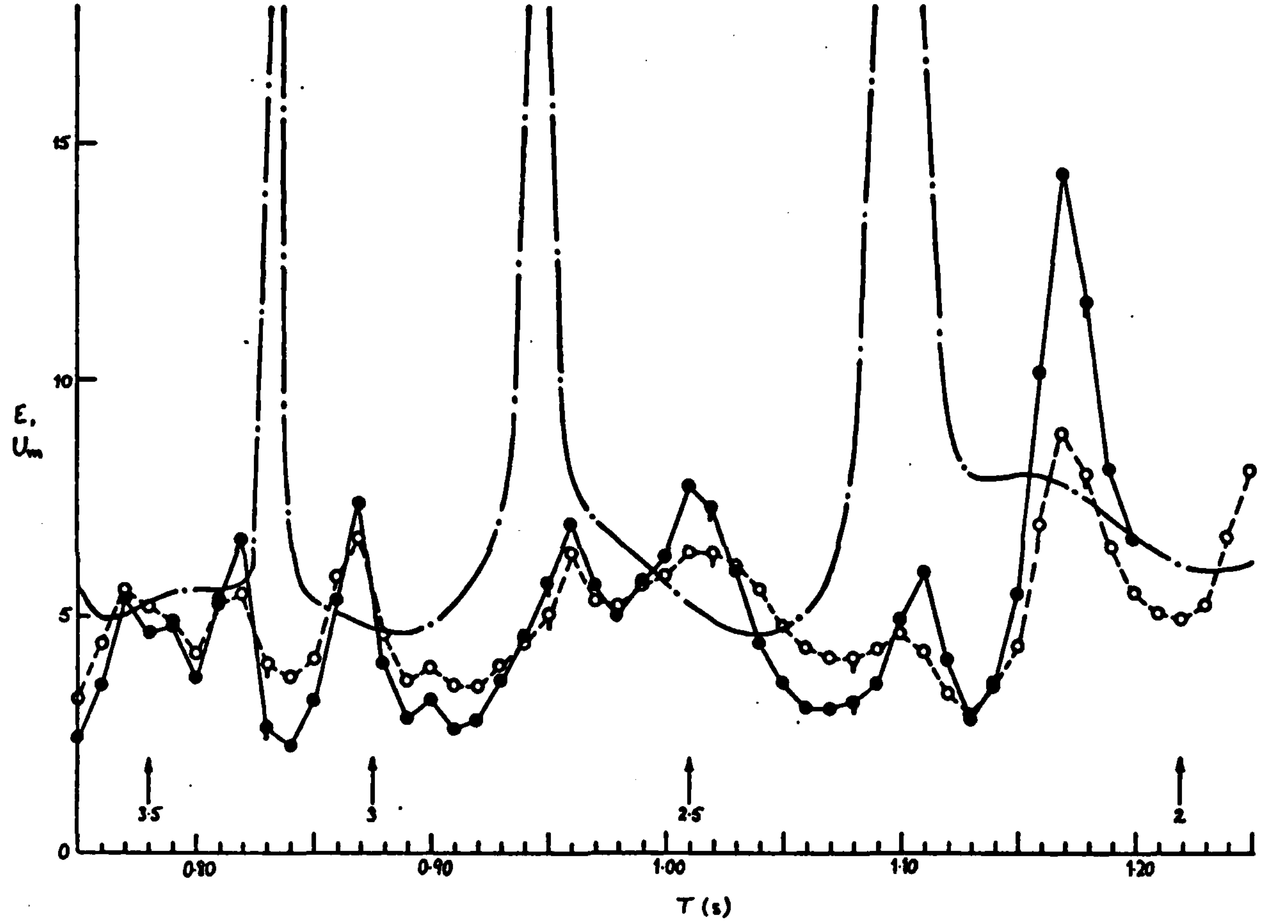

Figure 3. The functionals $U_m(T)$ (——o——) and $E(T)$ (——●——) of the wavefield $u(\underset{\sim}{x})$. The points marked ọ, ● are the results for the periods corresponding to the data of Figure 2. ——·—— is the theoretical curve for $E(T)$ according to Yamamuro-Renardy (1981); the theoretical values for the local maxima of U_m are 17.5 (near T = 0.83), 15.4 (near T = 0.95) and 11.6 (near T = 1.10).

PERIOD (s)	0.75	0.76	0.77	0.78	0.79	0.80	0.81	0.82	0.83	0.84	0.85	0.86	0.87	0.88	0.89	0.90	0.91
U_m	3.26	4.45	5.53	5.20	4.91	4.22	5.30	5.45	4.00	3.71	4.10	5.82	6.65	4.67	3.64	3.91	3.51
E	2.42	3.54	5.40	4.64	4.80	3.72	5.39	6.63	2.65	2.26	3.22	5.35	7.40	4.02	2.88	3.24	2.62
(I,J)	4,18	3,18	3,18	3,18	3,18	1,17	1,18	1,17	3,18	3,18	3,18	3,18	3,18	3,18	3,18	3,18	3,18
$\tilde{U}$	0.344	0.337	0.331	0.324	0.319	0.313	0.307	0.302	0.297	0.292	0.287	0.282	0.278	0.274	0.269	0.265	0.261
W	0.366	0.376	0.409	0.351	0.298	0.217	0.329	0.397	0.233	0.185	0.285	0.439	0.406	0.329	0.245	0.261	0.224

PERIOD (s)	0.92	0.93	0.94	0.95	0.96	0.97	0.98	0.99	1.00	1.01	1.02	1.03	1.04	1.05	1.06	1.07	1.08
U_m	3.51	3.92	4.46	5.02	6.34	5.31	5.21	5.62	5.87	6.38	6.32	6.07	5.55	4.78	4.35	4.16	4.12
E	2.81	3.62	4.56	5.68	7.48	5.65	5.00	5.75	6.24	7.75	7.30	5.96	4.46	3.59	3.07	3.03	3.19
(I,J)	3,18	3,18	3,18	3,18	3,18	3,18	2,18	2,18	2,18	2,18	2,18	2,18	2,18	2,18	2,18	2,18	2,18
$\tilde{U}$	0.257	0.254	0.250	0.246	0.243	0.240	0.236	0.233	0.230	0.227	0.224	0.221	0.219	0.216	0.213	0.211	0.208
W	0.258	0.285	0.290	0.226	0.232	0.244	0.232	0.227	0.249	0.284	0.290	0.236	0.243	0.222	0.192	0.145	0.139

PERIOD (s)	1.09	1.10	1.11	1.12	1.13	1.14	1.15	1.16	1.17	1.18	1.19	1.20	1.21	1.22	1.23	1.24	1.25
U_m^*	4.32	4.63	4.24	3.35	2.94	3.56	4.37	6.95	8.87	8.01	6.45	5.49	5.08	4.97	5.22	6.65	8.07
E	3.55	4.92	5.93	4.05	2.78	3.56	5.48	10.19	14.38	11.63	8.13	6.66	-	-	-	-	-
(I,J)	2,18	1,18	1,17	4,18	3,18	2,18	2,18	2,18	2,18	2,18	2,18	2,18	3,18	3,18	2,18	2,18	2,18
$\tilde{U}$	0.206	0.204	0.201	0.199	0.197	0.195	0.193	0.191	0.189	0.187	0.185	0.183	0.181	0.179	0.178	0.176	0.174
W	0.177	0.250	0.220	0.163	0.151	0.162	0.217	0.341	0.329	0.208	0.166	0.172	0.197	0.174	0.162	0.131	0.118

TABLE 2. Summary of the main results. The quoted values of U_m and E are those used for figure 3 and are measured relative to the wave 'amplitude' U (c f. §3.2(ii)); W (c f. §3.4) is the wave 'amplitude' used to scale the data for figure 7; (I,J) is the location of the mesh point at which $\max(U_{ij})$ was found. (Notes: (i) $\tilde{U}$ and W are amplitudes per unit stroke of the wavemaker; (ii) the empirical values for figure 7 are respectively $\tilde{U}/W$ and $(\tilde{U}/W)^2$ times the stated values for U_m and E.)

taken to be the (theoretical) amplitude of the wavefield generated in an 'ocean' of uniform depth D by a two-dimensional wavemaker oscillating at an amplitude S, as given by the classical (linear) theory of Havelock (1929) (or see, for example, Ursell et al. 1960). The actual values of U that were used are given below in table 2 where the main results are summarized.

It is seen in figure 3 that there are a number of similarities between the empirically determined values of U_m and E with, for example, their local maxima and minima occurring at approximately the same values of T. The largest observed values of U_m and E were recorded at T = 1.1700 and were respectively 8.87 and 14.4. This means that at the most dangerous location above the sill the waves were approximately nine times as large as would have been expected with no sill present. At other periods the maximum amplification was not so great, taking values of 4 or 5 with occasional peaks and troughs outside this range. The 'base' value for E was also around 4, (note that in the absence of the sill E would take a value of approximately $\frac{1}{2}\pi$), which is not greatly different from the 'base' levels predicted by the theory of Renardy. On the other hand, there is no evidence in this set of data for the dramatic resonances expected theoretically.

The graphs shown in figure 3 illustrate the importance of some of the experimental considerations described above. For example, the bandwidth of the 'resonance' near T = 0.833s is only about 50% greater than the increment in T used in the experiment and the need to maintain a fine control over the wavemaker period for a relatively large time span is evident. An explanation of the vertical arrows labelled 2, 2.5, 3, 3.5 is given below.

iii) <u>The wavefield at T = 1.1800s</u>

Because the most significant responses in this experiment would appear to have occurred at periods T between 1.16 and 1.19s, the wavefield at T = 1.1800s has been examined in some detail. The data from which the contour map of figure 2(e) was constructed is depicted in figure 4 where sections of the field $U(\underset{\sim}{x})$ for constant r and constant θ are given. To help interpret these data consider the representation of the wavefield above the sill used by Longuet-Higgins (1967), namely

$$U(r,\theta;\ t) = \sum_{m=-\infty}^{\infty} A_m J_m(\mu r)\ e^{i(m\theta+\sigma t)} ,$$

where the A_m's are complex constants, J_m is the Bessel function of the first kind of order m and μ is the real solution of $\mu \tanh(d\mu/a) = a\sigma^2/g$; g is the gravity constant. It is of interest to find the values for A_m in such a representation that would yield the best fit with the experimental data, through say a Fourier analysis of the data. But unfortunately no phase measurements were made in the experiment, and a direct analysis was not possible. Nevertheless an attempt has been made to 'fit' the amplitude data. Values for the (magnitudes) of the coefficients A_m given in table 1 correspond to coefficients that give a reasonably good description of the measurements. (The arguments leading to this choice are rather complex, but they are fully detailed in the thesis of Yamamuro-Renardy 1980). Also given in table 1 are the expected values from the Longuet-Higgins (1967) theory for a decomposition that realises an incident plane wave from the direction θ = 0.

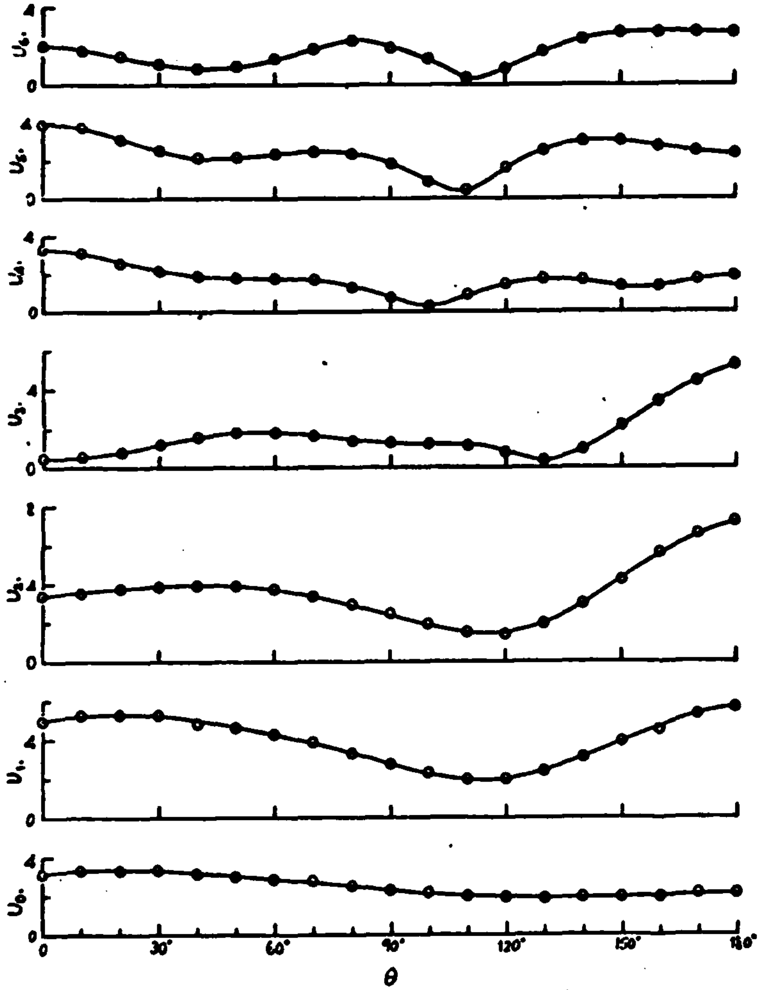

Figure 4a

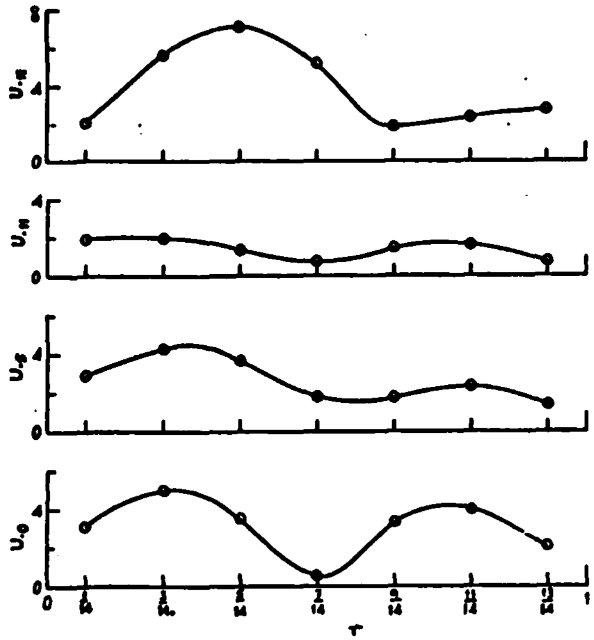

Figure 4b

Figure 4. Sections of the amplitude distribution $u(\underset{\sim}{x})$ depicted in Figure 2(e) for $T = 1.1800s$.

(a) The sections $r = \frac{1}{14}, \frac{3}{14}, \frac{5}{14}, \frac{7}{14}, \frac{9}{14}, \frac{11}{14}, \frac{13}{14}$;

(b) The sections $\theta = 0°, 60°, 120°, 180°$.

	$\lvert A_o \rvert$	$\lvert A_1 \rvert$	$\lvert A_2 \rvert$	$\lvert A_3 \rvert$	$\lvert A_4 \rvert$
Experiment	1.6	15.5	small	3.1	small
Theory	1.6	8.0	3.0	3.0	1.6

Table 1. Estimates of the magnitudes of the first four Fourier-Bessel coefficients A_m that give the 'best' description of the data of figure 4. The theoretical values are those for the theory of Longuet-Higgins (1967) at the same frequency.

This comparison suggests an almost complete absence of the $m = 2$ and $m = 4$ modes and the enhancement of the mode $m = 1$. Attempts to resolve these differences through nonlinear effects were unsuccessful, though the inclusion of a reflected wave from the beach behind the sill (in the direction $\theta = \pi$) did improve the agreement a little. Thus it would appear that the resolution of these differences, especially the large response of the mode $m = 1$, is unlikely to be achieved through an analysis based on the decomposition of a <u>plane</u> incident wave.

3.3 The wavefield in the 'ocean'

Some measurements were made of the wavefield in the region between the sill and the wavemaker to ascertain the level of any standing-wave motion in that zone. One such measurement was made along the line AB indicated in figure 1, wave amplitudes being recorded at seven different locations. Some examples of these results are given in figure 5, where the abscissa represents the distance from the wavemaker and U_o is the wave amplitude relative to the amplitude at the waterline of the (sinusoidal) displacement of the wavemaker. Also shown in the figure is the theoretical wavelength λ of the motion at the given period. The curves connecting the data points were drawn only as a possible guide to help correlate the data, but noting that with a two-dimensional reflected wavefield,

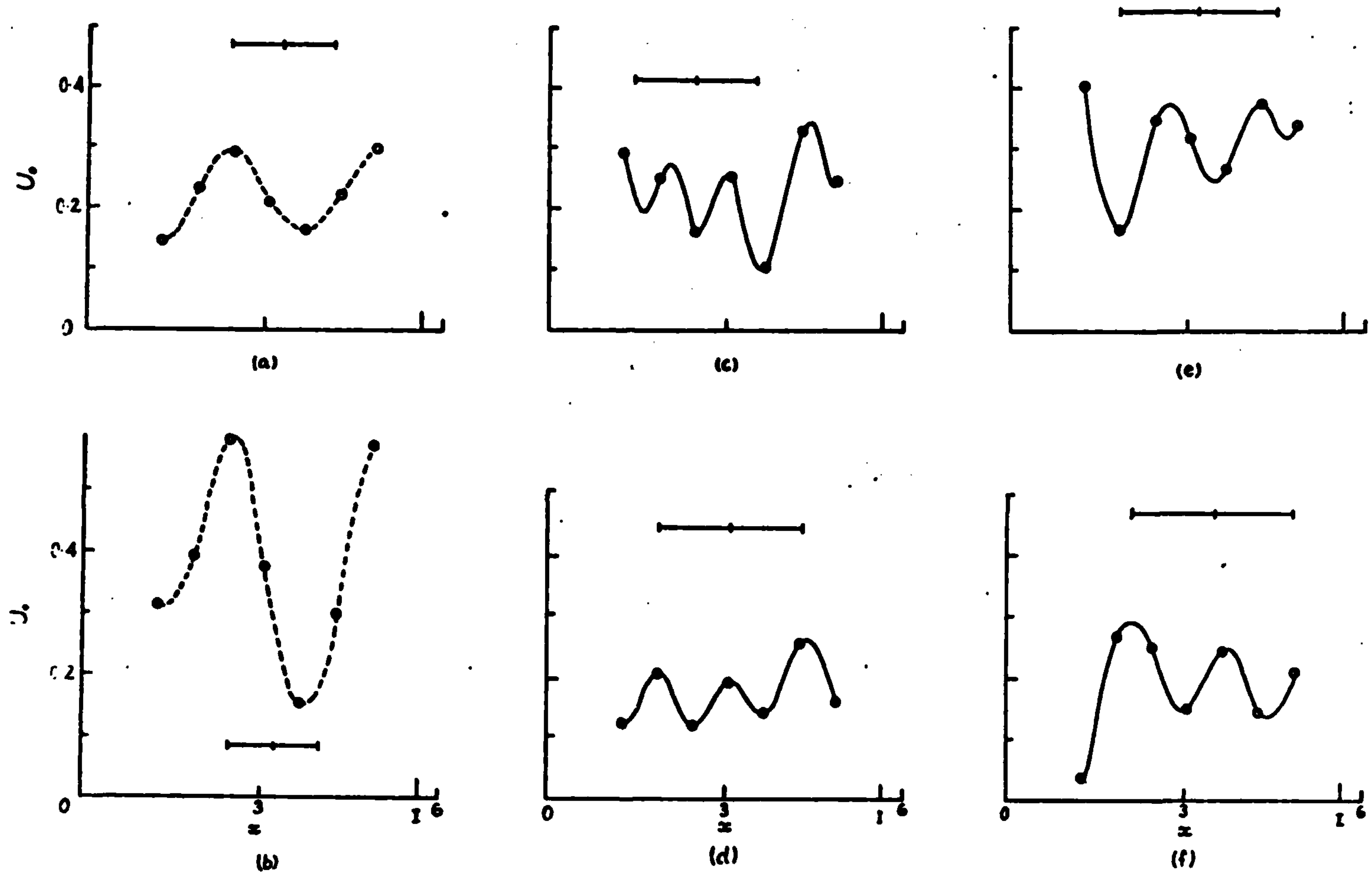

Figure 5. Wave amplitudes U_o between the wavemaker and the island, at the periods indicated. x is the distance from the wavemaker; I indicates the position of the front of the island; the horizontal line on each diagram corresponds to the wavelength.

U_o should undulate at a wavelength $\lambda/2$ (see, for example, Mahony & Pritchard 1980). (Since at the smaller values of T the measurements of U_o were each separated by about half a wavelength of the forced motion, the dashed curves have been drawn just to link the data points without respecting the anticipated $(\lambda/2)$ - undulations of U_o.)

The results shown in figure 5 indicate the presence of an appreciable standing-wave component of the wavefield, arising as a result of reflections from the submerged cylinder and the beaches. Since this standing-wave field could easily give rise to anomalous results should there be any tuning effects (e.g. between the wavemaker and the sill) we have indicated by the vertical arrows in figure 3 some of the periods at which such effects could have been prominent; shown near each arrow is the number of wavelengths between the wavemaker and the 'front' edge of the sill. The indicated periods are seen to coincide with local maxima of U_m and E, except for the one near $T = 1.22s$ which is near a local minimum.

A few measurements were also made along a line parallel to the wavemaker at a distance 56 cm ($\equiv 1.12a$) from the wavemaker to examine the degree of nonuniformity in this direction. The results, which are given in figure 6, confirm the complex, three-dimensional nature of the wave scattering from the sill and the beaches already suggested by the results of figure 5. Thus, in view of the apparently complicated structure of the wavefield in the 'ocean' beyond the sill, it was decided to make an alternative assessment of the results depicted in figure 3.

3.4 A rescaling of the data

A possible explanation of the result that none of the theoretical resonances were apparent in the results of figure 3 is that, because of

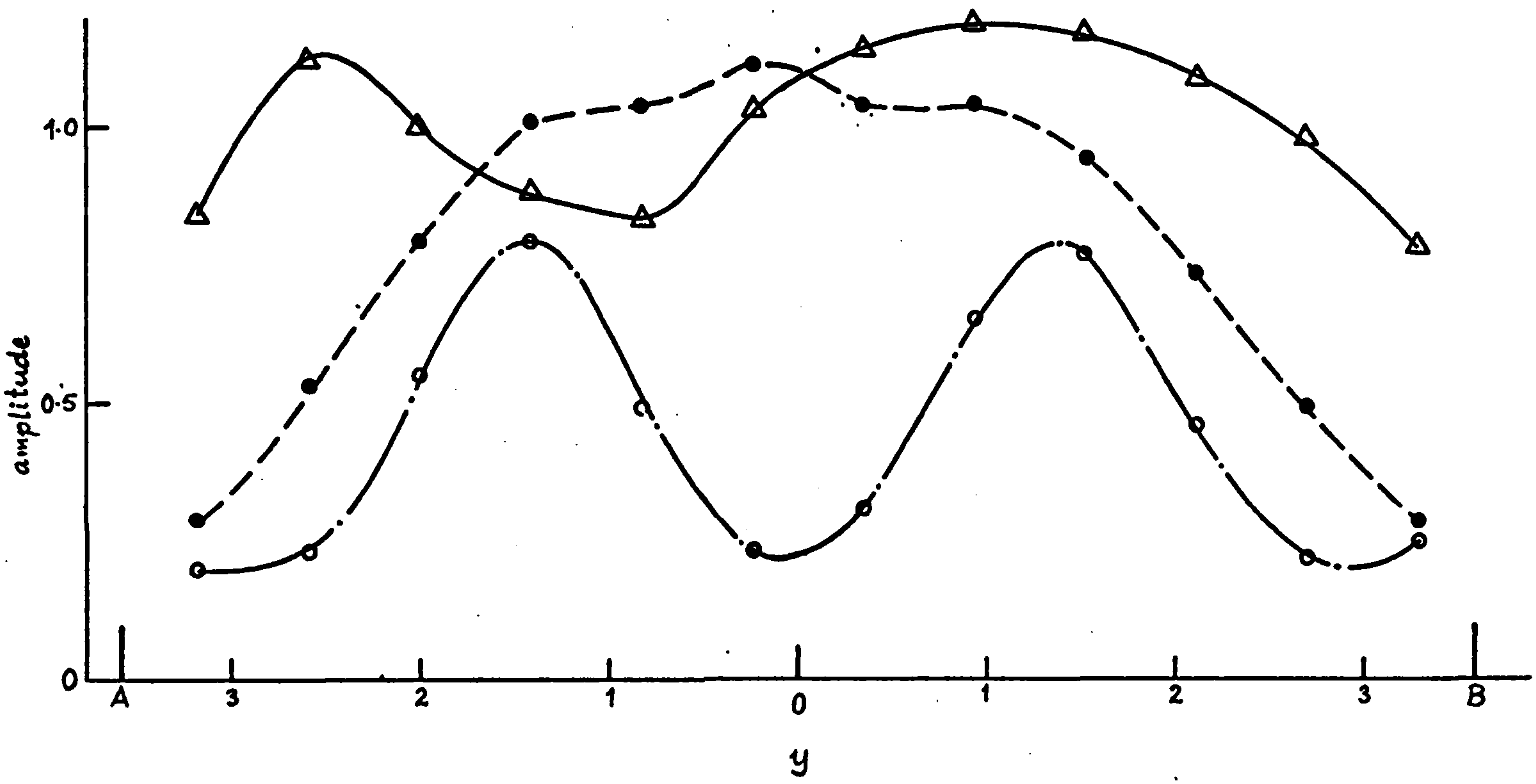

Figure 6. Wave amplitudes measured along a line parallel to the wavemaker at a distance 1.12a from the wavemaker. y is the distance from the centreline of the tank; the amplitude scale is arbitrary; A and B represent the positions of the left and right-hand sides of the wavemaker. ———: T = 0.8015s; — — —: T = 1.0015s; —·—: T = 1.2015s.

the appreciable standing-wave component of the field in the 'ocean', the 'estimate' for the effective incident wave used there was grossly inaccurate. We have, therefore, tried to assess the importance of this feature by using data of the kind shown in figure 5 to provide an empirical estimate W, say, of the incident wave field from the 'mean' wave amplitude in the zone between the wavemaker and the sill (c f. Mahony & Pritchard 1980 for a description of this kind of partitioning). For example, at period T = 1.22s for the data shown in figure 5(f), W was chosen to be 0.174. All the values for W thus chosen are given in table 2, together with the values of $\tilde{U}$ used to construct figure 3. The result of using the estimate W for the incident wave levels is shown in figure 7, where U_m and E have been replotted. Here the four largest maxima, in order of decreasing magnitude, occurred for E at T = 1.19, 0.96, 0.80 and 1.08s and for U_m at T = 1.18, 0.96, 1.08 and 0.80s. Note that the overall maximum for figure 3 was near T = 1.17s, so that the rescaling has not seen the elimination of the large response near this period. But while some more prominant local maxima are evident in figure 7, there is still no indication of the very large responses expected theoretically.

4. ASSESSMENT & DISCUSSION

The results obtained from these experiments do not show any indication of the resonances expected theoretically when a train of plane waves passes over a submerged cylindrical island situated in an otherwise uniform ocean. Some of the possible reasons for this disparity have been examined in §3 where, in particular, attempts have been made to assess the laboratory experiment in terms of the hypothetical problem studied theoretically.

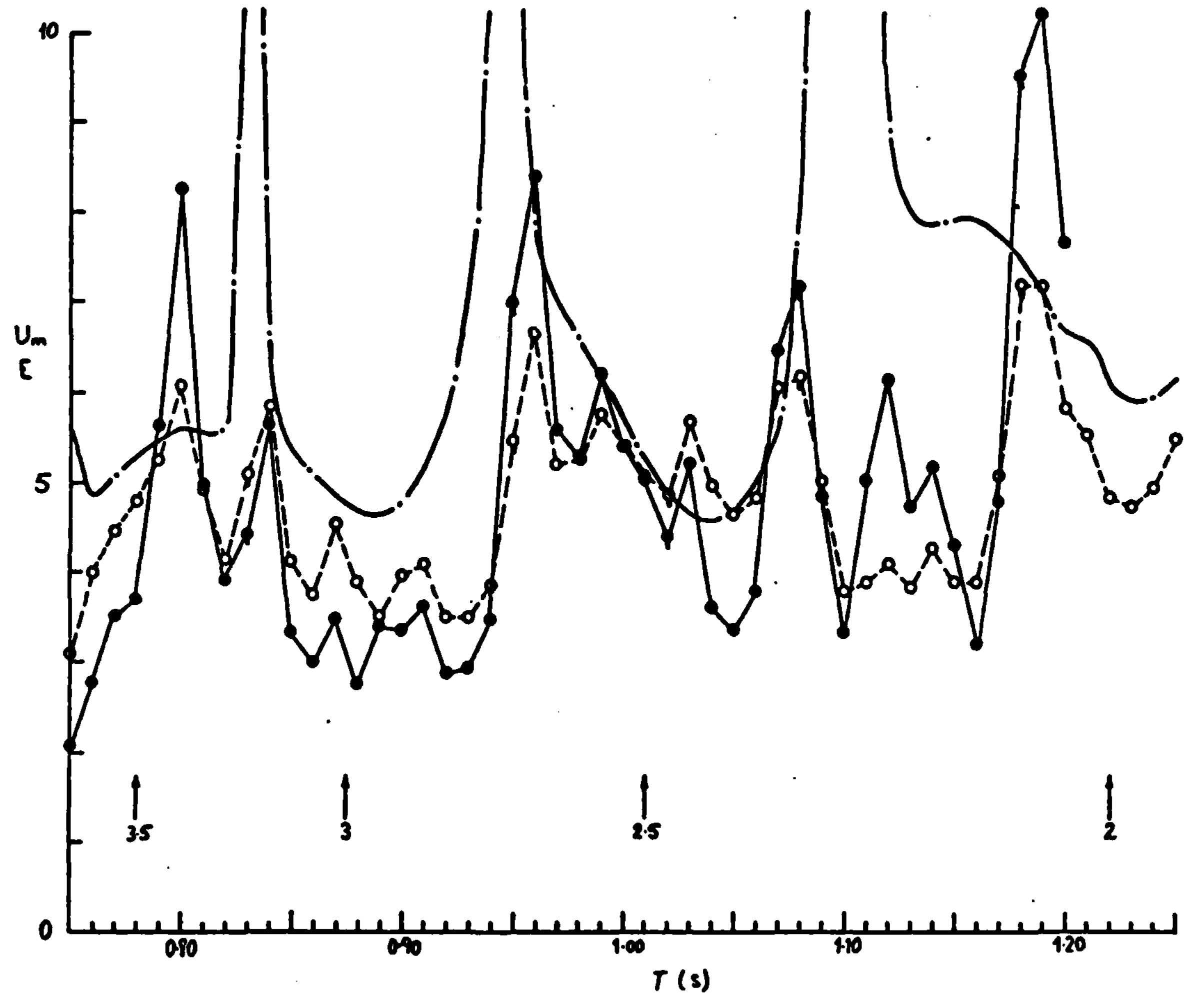

Figure 7. The quantities $U_m(T)$ (–○–) and $E(T)$ (—●—) measured relative to the amplitude scale W (c f. the text and Table 2). The points marked ǫ, ● are the results for the periods corresponding to the data of Figure 2. —·— is the theoretical curve for $E(T)$ according to Yamamuro-Renardy (1981). The meaning to be attached to the numbered arrows is given in the text.

The elimination of unwanted motions in water wave experiments is usually very difficult (e.g. see Bona, Pritchard & Scott, 1981) and the present experiment has proved to be no exception. Thus, a particular difficulty arising in this study has been that of defining a reference wave amplitude against which the response over the sill could be scaled. Two reference amplitudes were considered in §3 : one is related directly to the amplitude of the wavemaker used to force the motions (through classical wavemaker theory) and the other was determined from wave measurements in the tank. Each of these procedures is inadequate in one way or another (with regard to their relation to the theoretical situation). Nevertheless, at periods remote from the theoretical 'resonances', the measured values of the quantities U_m and E (c.f. figures 3, 7) were in reasonably good agreement with the theoretical predictions of the model calculation in the sense that the discrepancies between the theory and the experiment could be accounted for by a change of the reference-wave amplitude of not more than about 30%. Near the resonant periods, however, the estimates for the reference-wave amplitude would have to be changed by a factor of over 3 to account for the observed discrepancies, and such large factors should have been apparent in the experimental results. An important aspect of the discussion of §3, and bearing on the above issues, was that the response over the sill was not entirely compatible with the decomposition of an incident plane wave from infinity and this would appear to present the main source of difficulty in interpreting the experimental situation in terms of the hypothetical model.

A salient feature of the 'resonance' phenomenon under consideration is that leakage rates of certain eigenvalues become small near particular frequencies, enabling the build-up of the corresponding wavemode to unusually large amplitudes. But, in consequence of the small leakage rate, the coupling between the external, incident wavefield and that over the island is very weak, so, for significant amounts of energy to be transferred to the field above the island, the exterior conditions must be held steady for a considerable time.

The design of the wavemaker drive, together with the very close control maintained over the frequency input to it, ensured that phase drifting was unimportant here. Also, the very small amplitudes (and surface slopes) employed in this study would appear to have precluded the possibility that nonlinear effects could have overwhelmed the resonance. On the other hand, it is more difficult to assess the importance of surface contamination on the motions. We do know that if the surface became unduly contaminated the wavefield was influenced significantly; but the fact that the general level of response was in broad agreement with the levels anticipated theoretically (c.f figures 3,7) suggests that the effects of surface contamination was not a dominating feature of the experiment. According to Renardy (1981) dissipative effects on the rigid boundaries should not have been important.

Thus, although the experiment did not apparently realise a good representation of the hypothetical wavefield needed for the theoretical model, it is nontheless surprising that there were no manifestations whatsoever of the dramatic resonances predicted theoretically, and an explanation of the observed response would therefore be desirable. Because the beaches used in the experiments should have been fairly efficient absorbers of energy, such an explanation might, in the first place, be sought from a model with a long wavemaker generating waves near an island in a semi-infinite ocean. We believe this experiment might present a reasonably critical test for numerical schemes for water-wave problems of this kind and we have therefore included (see table 2) a summary of the main experimental results.

Finally we should like to make some observations concerning the siting of structures on submerged ridges of this kind. While, in the above study, the theory indicates that at certain 'resonant' frequencies unusually large wave responses can prevail, both the theory and the experiments indicate that, at certain dangerous locations on the sill the waves are magnified by a factor of about five times that of the prevailing ocean swell, irrespective of the frequency. Under moderate conditions at sea this might not be a serious problem, but with a heavy swell running an amplification by five times could be catastrophic. Fortunately the very dangerous locations seem to be fairly insensitive to frequency, for a given prevailing direction; by contrast, there are apparently also quiescent regions, again fairly insensitive to frequency, where the amplication is less than one.

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A description is given of some experiments made to examine the possible trapping of surface waves by a submerged cylindrical sill in an otherwise uniform ocean. Theories by Longuet-Higgins (1967) and Renardy (1981) indicate that, at certain dangerous frequencies, nearly resonant conditions my obtain, leading to unusually large wavefields over the sill. may

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ABSTRACT (cont.)

These 'resonances' have been sought experimentally by examining two functionals of the wavefield over the sill to see if they should become unusually large near certain of the 'dangerous' frequencies. The results show no manifestation whatsoever of the resonances. Possible reasons for the discrepancies are considered.

It has, however, been observed that, at certain locations over the sill, the waves were amplified by a factor of four or five times their magnitude in the 'ocean', irrespective of the frequency in a certain interval. This could be an important consideration for the siting of rigs on prominent topographic features of the ocean bed.